B.A./B.Sc. 1st Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH1CC-II

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Find all the values of $(-i)^{\frac{3}{4}}$.

(b) Solve the equation $x^3 + 6x^2 + 11x + 6 = 0$, the roots are in arithmetic progression. $a-d,\ a,\ a+d$

(c) If n is a positive integer, then prove that $\left(1+\frac{1}{n+1}\right)^{n+1} > \left(1+\frac{1}{n}\right)^{n}$.

(d) Define $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = |x|x, \forall x \in \mathbb{R}$. Examine if f is one-to-one. 11

(e) Define an orthogonal matrix. Show that any orthogonal matrix is non-singular.

(f) Find the rank of the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 2 \\ 1 & 4 & 0 \end{pmatrix}$.

(g) Examine if $W = \{(x,y,z) \in \mathbb{R}^3 : x^2 - y^2 = z^2\}$ is a subspace of $\mathbb{R}^3$.

(h) State Cayley-Hamilton theorem. Verify this for $A = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$.

(i) If A is an $n \times n$ matrix over $\mathbb{R}$, show that the product of eigenvalues of A is $\det(A)$.

(j) Show that the real matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ has no real eigenvalue.

(k) Find the remainder when $1! + 2! + 3! + \ldots + 50!$ is divided by 15.

(l) Let a and b be two positive integers such that gcd $(a, b) = 1$. Find gcd $(a + b, a - b)$.

(m) Give an example a binary relation which is reflexive but not symmetric and transitive. Justify your answer.

(n) Find a basis for the vector space $\mathbb{R}^3$ that contains the vectors (1, 2, 0) and (1, 3, 1).

(o) Apply Descarte's rule of signs to examine the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$.

2. Answer *any four* questions: 5×4=20

(a) State and prove De Moivre's theorem. 1+4

(b) If x, y, z are positive reals and $x + y + z = 1$, then prove that $8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}$.

(c) If P is an orthogonal matrix with $\det(P) = -1$, then show that -1 is an eigenvalue of P.

(d) Solve the equation $x^3 - 3x - 1 = 0$ by Cardan's method.

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(e) (i) Calculate Sturm's functions and locate the position of the real roots of the equation $x^3 - 3x - 1 = 0$.

(ii) Let $f: A \to B$ be an onto mapping and S, T be subsets of B. Then prove that $S \subseteq T \Rightarrow f^{-1}(S) \subseteq f^{-1}(T)$. 3+2

(f) Show that $W = \{(x, y, z) \in \mathbb{R}^3 : x + y - z = 0\}$ is subspace of $\mathbb{R}^3$. Find also the dimension of W. 2+3

3. Answer *any two* questions: 10×2=20

(a) (i) Find all positive integral solutions of $9x + 7y = 200$.

(ii) Solve the system of linear equations, of possible:

$$x + 2y + z - 3w = 1$$

$$2x + 4y + 3z + w = 3$$

$$3x + 6y + 4z - 2w = 5$$

5+5

(b) (i) Show that the product of all the values of $(1 + \sqrt{3}i)^{\frac{3}{4}}$ is 8.

(ii) Solve the equation $x^4 + 12x - 5 = 0$ by Ferrari's method. 5+5

(c) (i) A is a 3×3 real matrix having the eigenvalues 5, 2, 2. The eigenvector of A corresponding to the eigenvalues 5 and 2 are $c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $c \neq 0$ and $c\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + d\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, $(c, d) \neq (0, 0)$ respectively. Find the matrix A.

(ii) If every non-zero vector in $\mathbb{R}^n$ is an eigenvector of a real $n \times n$ matrix A, then prove that A is a scalar matrix. 5+5

(d) (i) A relation R is defined on the set $\mathbb{Z}$ by "$a\ R\ b$ iff $ab \geq 0$" for a, $b \in \mathbb{Z}$. Examine if R is (I) reflexive, (II) Symmetric, (III) Transitive.

(ii) Prove that a necessary and sufficient condition for a non-homogeneous system $AX = B$ to be consistent is rank A = rank $\bar{A}$, where $\bar{A}$ is the augmented matrix of the system. 5+5

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whe $a = -2$

$d^2 = \pm 1$